Adı-Soyadı:

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MATRİSLER TEORİSİ ARASINAV SORULARI

1. $B=\begin{bmatrix}1 & 1 & -2 & 4\\ 2 & 2 & -3 & 1\\ 3 & 3 & -4 & -2\end{bmatrix}$ matrisinin rankını bulunuz.

2. $\vec{v_1}=(1,2,0,1)$, $\vec{v_2}=(0,-1,2,1)$, $\vec{v_3}=(2,1,-3,0)$, $\vec{v_4}=(-1,0,1,-2)$ vektörlerinin lineer bağımlılık-bağımsızlık durumunu inceleyiniz.

3. Ortogonal matris, invers matris, involutif matris, nilpotent matris tanımlarını yapınız.

4. $A=\begin{bmatrix}3 & 5\\ 1 & 0\end{bmatrix}$ matrisini çarpanlarına ayırınız.

5. A ve B regüler kare matrisleri için $AB=A$ ve $BA=B$ olsun. A matrisinin transpozunun idempotent B matrisinin birim matris olduğunu gösteriniz.

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CEVAPLAR

1) $B=\begin{array}{l}\begin{bmatrix}1 & 1 & -2 & 4\\ 2 & 2 & -3 & 1\\ 3 & 3 & -4 & -2\end{bmatrix}\end{array}\begin{array}{l}\alpha_1\\ \alpha_2\\ \alpha_3\end{array}$

$E_1: \alpha_2 \to \alpha_2 - 2\alpha_1$
$\alpha_3 \to \alpha_3 - 3\alpha_1$

$E_2: \alpha_1 \to \alpha_1 + 2\alpha$
$\alpha_3 \to \alpha_3 - 2\alpha$

$\overset{E_1}{\approx}\begin{bmatrix}1 & 1 & -2 & 4\\ 0 & 0 & 1 & -7\\ 0 & 0 & 2 & -14\end{bmatrix} \overset{E_2}{\approx}\begin{bmatrix}1 & 1 & 0 & -10\\ 0 & 0 & 1 & -7\\ 0 & 0 & 0 & 0\end{bmatrix}=R$

$\text{rank} R = 2 \qquad R \approx A \quad \Rightarrow \text{rank} A = 2$

2) $A=\begin{array}{c|c} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 2 & 1 \\ 2 & 1 & -3 & 0 \\ -1 & 0 & 1 & -2 \end{bmatrix} & \begin{matrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{matrix} \end{array}$

$A \approx I$ ise $\{v_i\}$, $1 \le i \le 4$ sistemi lineer bağımsız.

$\varepsilon_1: \alpha_3 \to \alpha_3 - 2\alpha_1$
$\quad \alpha_4 \to \alpha_4 + \alpha_1$

$\varepsilon_2: \alpha_2 \to -\alpha_2$

$\varepsilon_3: \alpha_1 \to \alpha_1 - 2\alpha_2$
$\quad \alpha_3 \to \alpha_3 + 3\alpha_2$
$\quad \alpha_4 \to \alpha_4 - 2\alpha_2$

$$A \overset{\varepsilon_1}{\approx} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & -3 & -3 & 0 \\ 0 & 2 & 1 & -1 \end{bmatrix} \overset{\varepsilon_2}{\approx} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & -3 & -3 & 0 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

$$\overset{\varepsilon_3}{\approx} \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & -9 & -3 \\ 0 & 0 & 5 & 1 \end{bmatrix}$$

$\varepsilon_4: \alpha_3 \to -\frac{1}{3}\alpha_3$

$$\overset{\varepsilon_4}{\approx} \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 5 & 1 \end{bmatrix}$$

$\varepsilon_5: \alpha_4 \to \alpha_4 - 2\alpha_3$

$\varepsilon_6: \alpha_4 \to -\alpha_4$
$\quad \alpha_4 \leftrightarrow \alpha_3$

$$\overset{\varepsilon_5}{\approx} \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \overset{\varepsilon_6}{\approx} \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

$\varepsilon_7: \alpha_1 \to \alpha_1 - 4\alpha_3$
$\quad \alpha_2 \to \alpha_2 + 2\alpha_3$
$\quad \alpha_4 \to \alpha_4 - 3\alpha_3$

$\varepsilon_8: \alpha_4 \to -\frac{1}{2}\alpha_4$

$$\overset{\varepsilon_7}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \overset{\varepsilon_8}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overset{\varepsilon_9}{\approx} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$A \approx I \Rightarrow \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ sistemi lineer bağımsızdır.

3) Ders Notları

4) $A = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix} \overset{\varepsilon_1}{\approx} \begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix} \overset{\varepsilon_2}{\approx} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \overset{\varepsilon_3}{\approx} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R$

$\varepsilon_1 : \alpha_1 \longleftrightarrow \alpha_2$ $\qquad \Rightarrow \varepsilon_1^{-1} : \alpha_1 \longleftrightarrow \alpha_2$

$\varepsilon_2 : \alpha_2 \to \alpha_2 - 3\alpha_1$ $\qquad \varepsilon_2^{-1} : \alpha_2 \longrightarrow \alpha_2 + 3\alpha_1$

$\varepsilon_3 : \alpha_3 \to \frac{1}{5}\alpha_2$ $\qquad \varepsilon_3^{-1} : \alpha_2 \to 5\alpha_2$

$$I = \begin{array}{cc} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} \end{array}$$

$\varepsilon_3(\varepsilon_2(\varepsilon_1(A))) = R$

$\varepsilon_3(I)\,\varepsilon_2(I)\,\varepsilon_1(I)\,A = R$

$A = \varepsilon_1^{-1}(I)\,\varepsilon_2^{-1}(I)\,\varepsilon_3^{-1}(I)\,R$

$\varepsilon_1^{-1}(I) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\qquad \varepsilon_2^{-1}(I) = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $\qquad \varepsilon_3^{-1}(I) = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5) Ders notları